Chapter 6
Normal Probability Distributions

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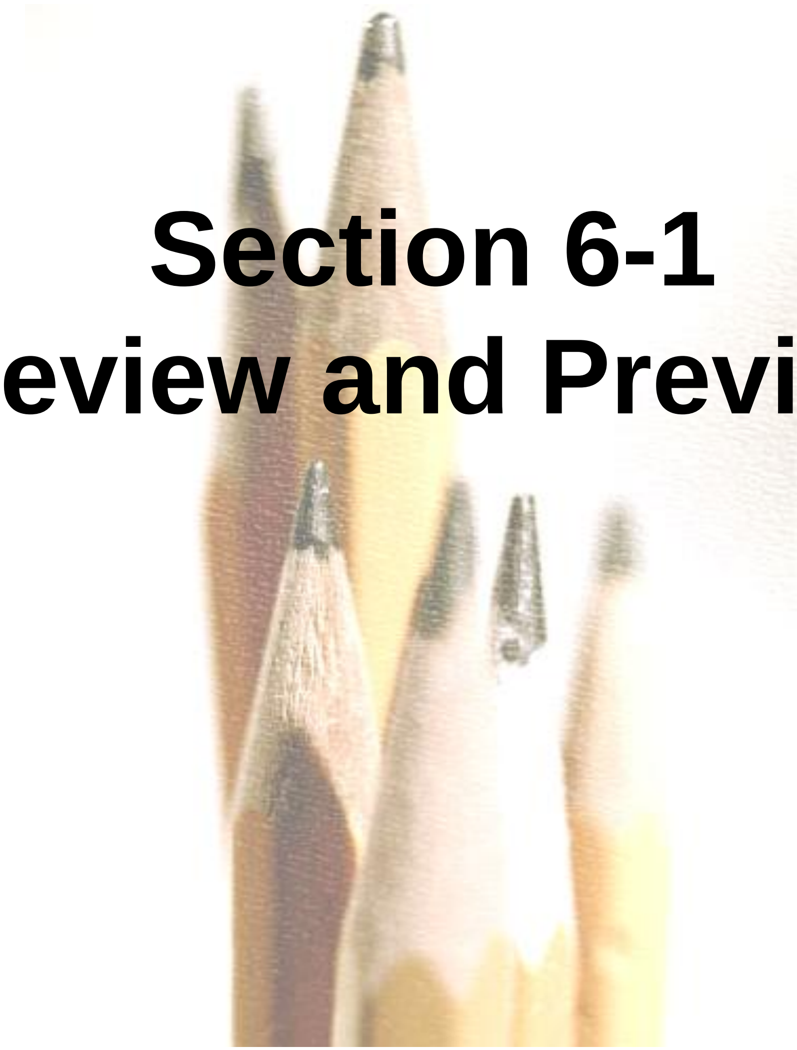
Section 6-1
Review and Preview



Review

- Chapter 2: Distribution of data
- Chapter 3: Measures of data sets, including measures of center and variation
- Chapter 4: Principles of probability
- Chapter 5: Discrete probability distributions

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Preview

Chapter focus is on:

- **Continuous random variables**
- **Normal distributions**

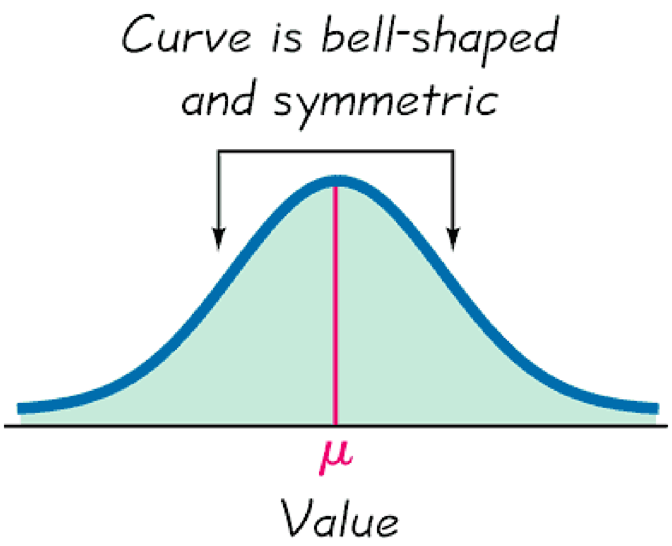

Figure 6-1

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

Formula 6-1

Distribution determined by fixed values of mean and standard deviation

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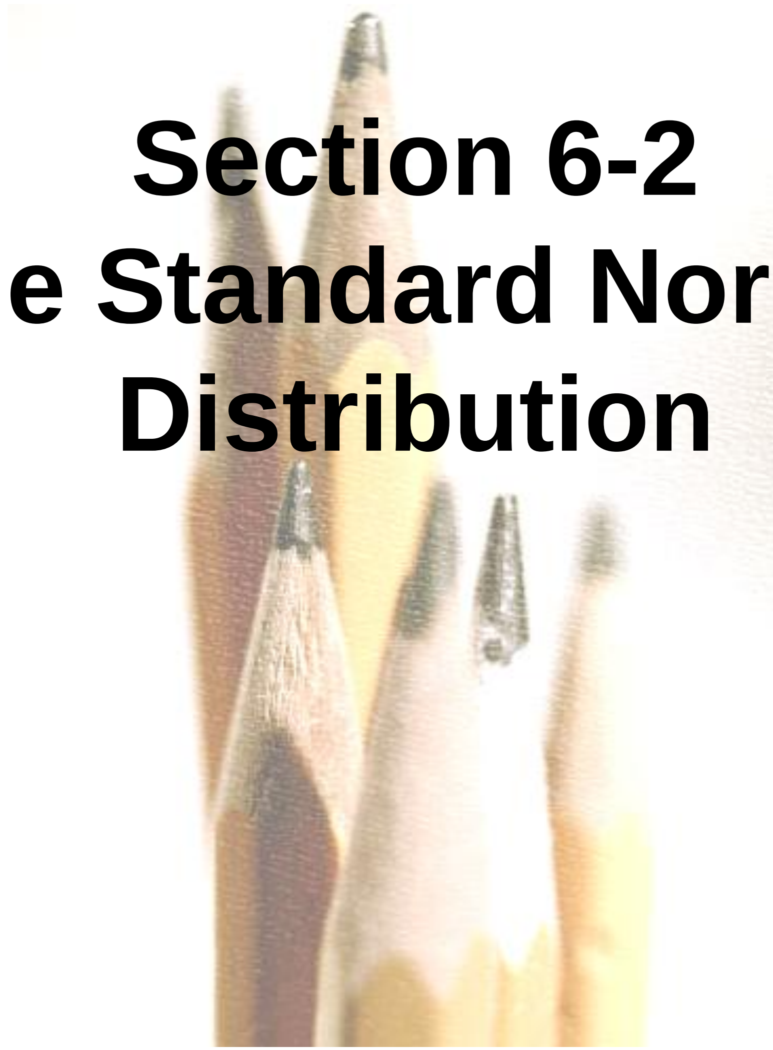
Section 6-2
The Standard Normal Distribution



Key Concept

This section presents the *standard normal distribution* which has three properties:

1. It's graph is bell-shaped.

2. It's mean is equal to 0 ($\mu = 0$).

3. It's standard deviation is equal to 1 ($\sigma = 1$).

Develop the skill to find areas (or probabilities or relative frequencies) corresponding to various regions under the graph of the standard normal distribution. Find *z*-scores that correspond to area under the graph.

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Uniform Distribution

A continuous random variable has a uniform distribution if its values are spread evenly over the range of probabilities. The graph of a uniform distribution results in a rectangular shape.

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Density Curve

A **density curve** is the graph of a continuous probability distribution. It must satisfy the following properties:

1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the *x*-axis.)

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Area and Probability

Because the total area under the density curve is equal to 1, there is a correspondence between *area* and *probability*.

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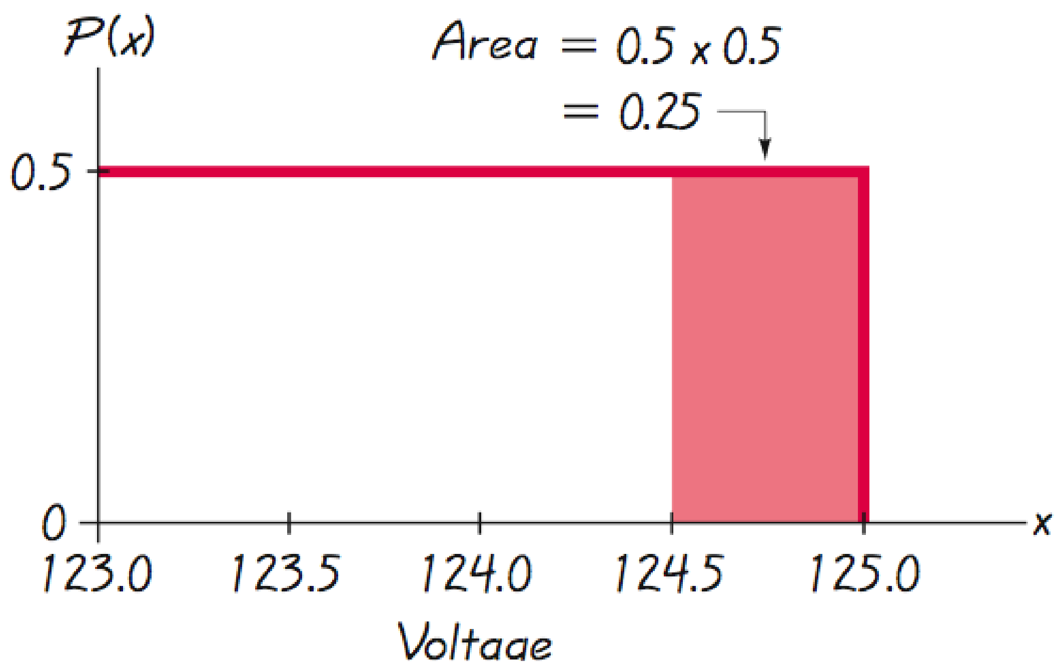
Using Area to Find Probability

Given the uniform distribution illustrated, find the probability that a randomly selected voltage level is greater than 124.5 volts.



Shaded area represents voltage levels greater than 124.5 volts. Correspondence between area and probability: 0.25.

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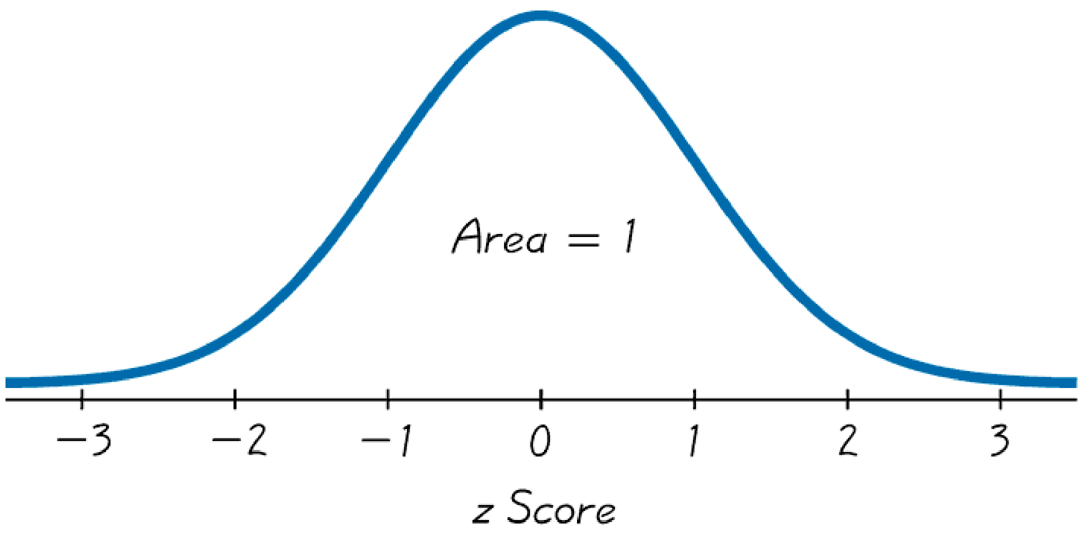
Standard Normal Distribution

The **standard normal distribution** is a normal probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.



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Finding Probabilities When Given *z*-scores

- **Table A-2 (in Appendix A)**
- **Formulas and Tables insert card**
- **Find areas for many different regions**

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Finding Probabilities – Other Methods

- STATDISK
- Minitab
- Excel
- TI-83/84 Plus

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Methods for Finding Normal Distribution Areas

Table A-2, STATDISK, Minitab, Excel

Gives the cumulative area from the left up to a vertical line above a specific value of *z*.

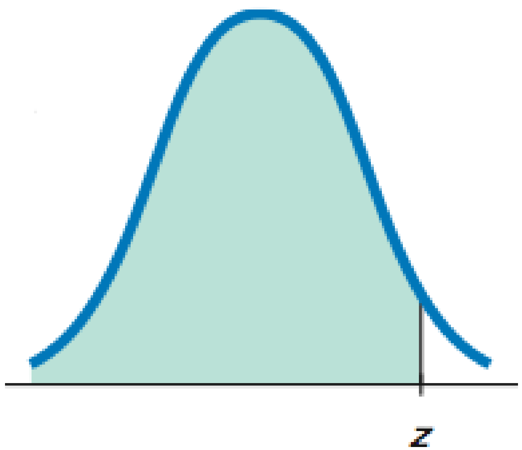

Table A-2 The procedure for using Table A-2 is described in the text.

STATDISK Select **Analysis, Probability Distributions, Normal Distribution.** Enter the *z* value, then click on **Evaluate.**

MINITAB Select **Calc, Probability Distributions, Normal.** In the dialog box, select **Cumulative Probability, Input Constant.**

EXCEL Select **fx, Statistical, NORMDIST.** In the dialog box, enter the value and mean, the standard deviation, and "true."

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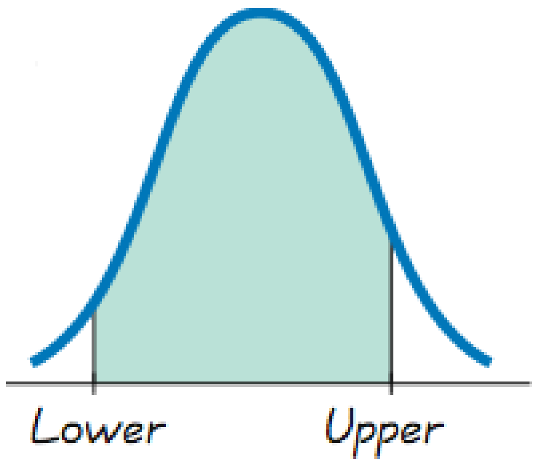
Methods for Finding Normal Distribution Areas

TI-83/84 Plus Calculator

Gives area bounded on the left and bounded on the right by vertical lines above any specific values.



TI-83/84 Press 2ND VARS **[2: normal cdf ( ],** then enter the two *z* scores separated by a comma, as in (left *z* score, right *z* score).

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Table A-2

TABLE A-2 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	* .0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

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Using Table A-2

1. **It is designed only for the *standard* normal distribution, which has a mean of 0 and a standard deviation of 1.**
2. **It is on two pages, with one page for *negative* *z*-scores and the other page for *positive* *z*-scores.**
3. **Each value in the body of the table is a *cumulative area from the left* up to a vertical boundary above a specific *z*-score.**

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Using Table A-2

4. **When working with a graph, avoid confusion between *z*-scores and areas.**

   ***z* Score**

   Distance along horizontal scale of the standard normal distribution; refer to the <u>leftmost column and top row</u> of Table A-2.

   Area

   Region under the curve; refer to the values in the <u>body</u> of Table A-2.

5. **The part of the z-score denoting hundredths is found across the top.**

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Example - Thermometers

The Precision Scientific Instrument Company manufactures thermometers that are supposed to give readings of 0ºC at the freezing point of water. Tests on a large sample of these instruments reveal that at the freezing point of water, some thermometers give readings below 0º (denoted by negative numbers) and some give readings above 0º (denoted by positive numbers). Assume that the mean reading is 0ºC and the standard deviation of the readings is 1.00ºC. Also assume that the readings are normally distributed. If one thermometer is randomly selected, find the probability that, at the freezing point of water, the reading is less than 1.27º.

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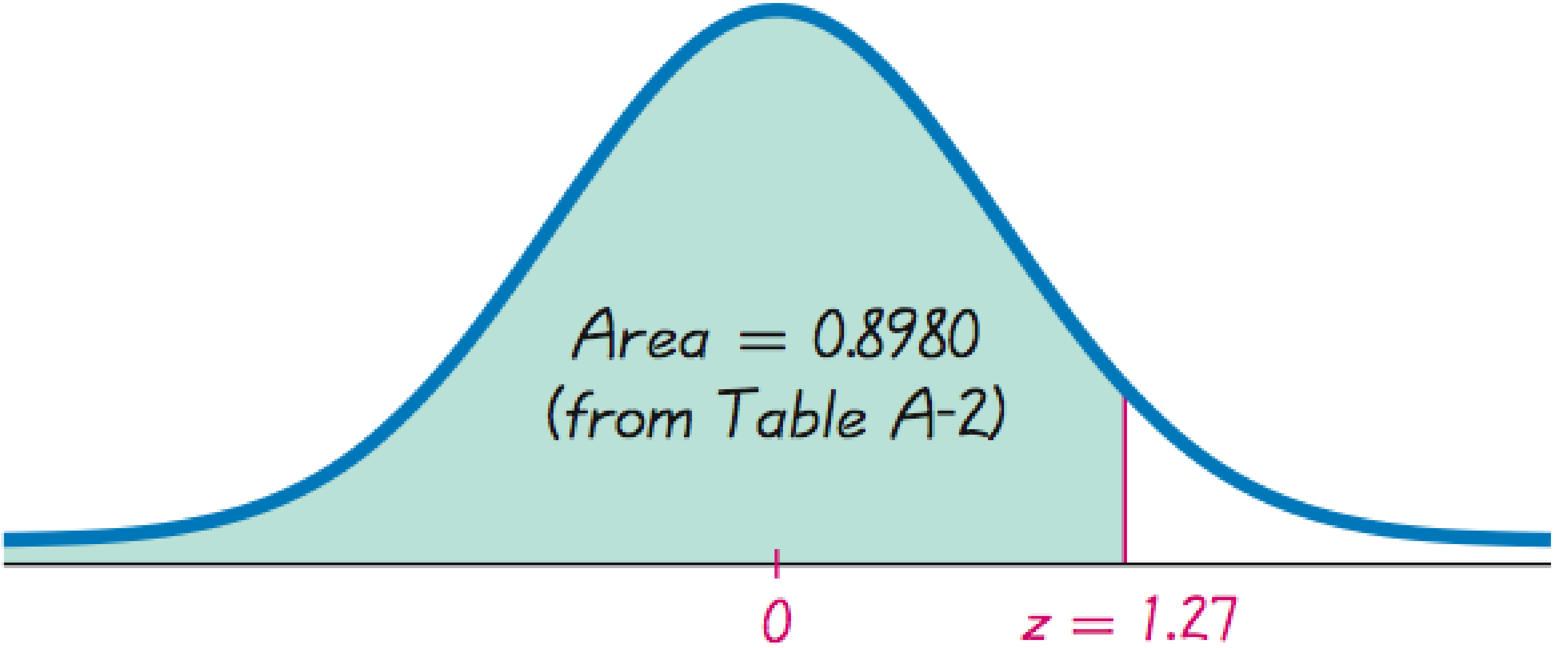
Example - (Continued)

$P(z < 1.27) =$



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Look at Table A-2

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292

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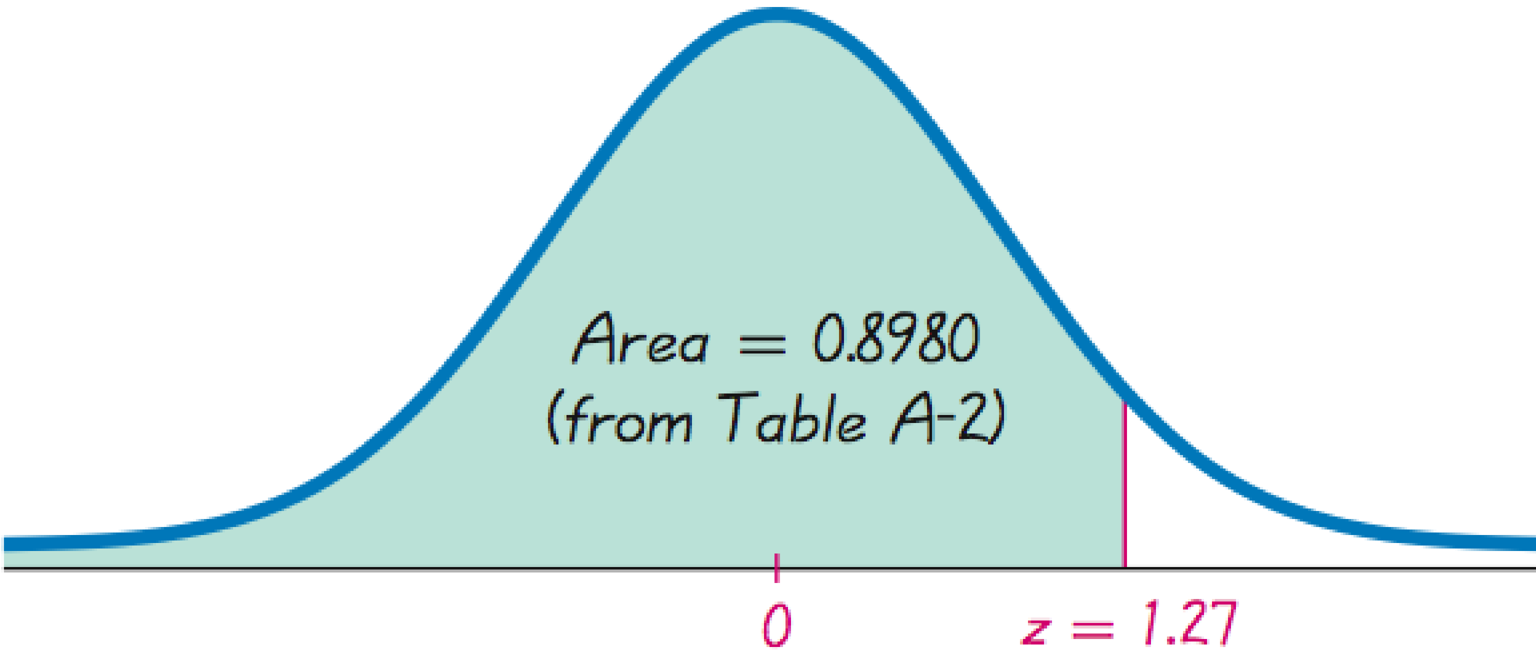
Example - cont

P (z < 1.27) = 0.8980



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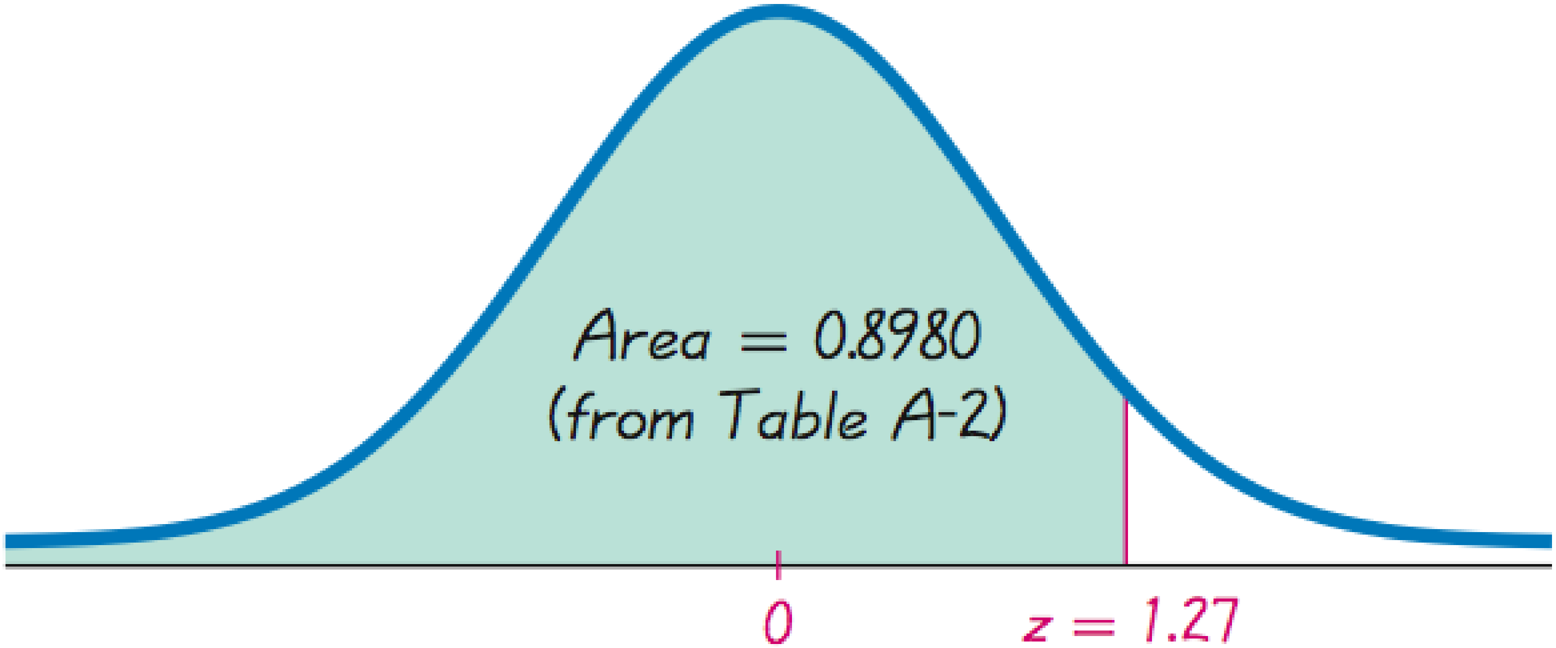
Example - cont

P (z < 1.27) = 0.8980



The *probability* of randomly selecting a thermometer with a reading less than 1.27º is 0.8980.

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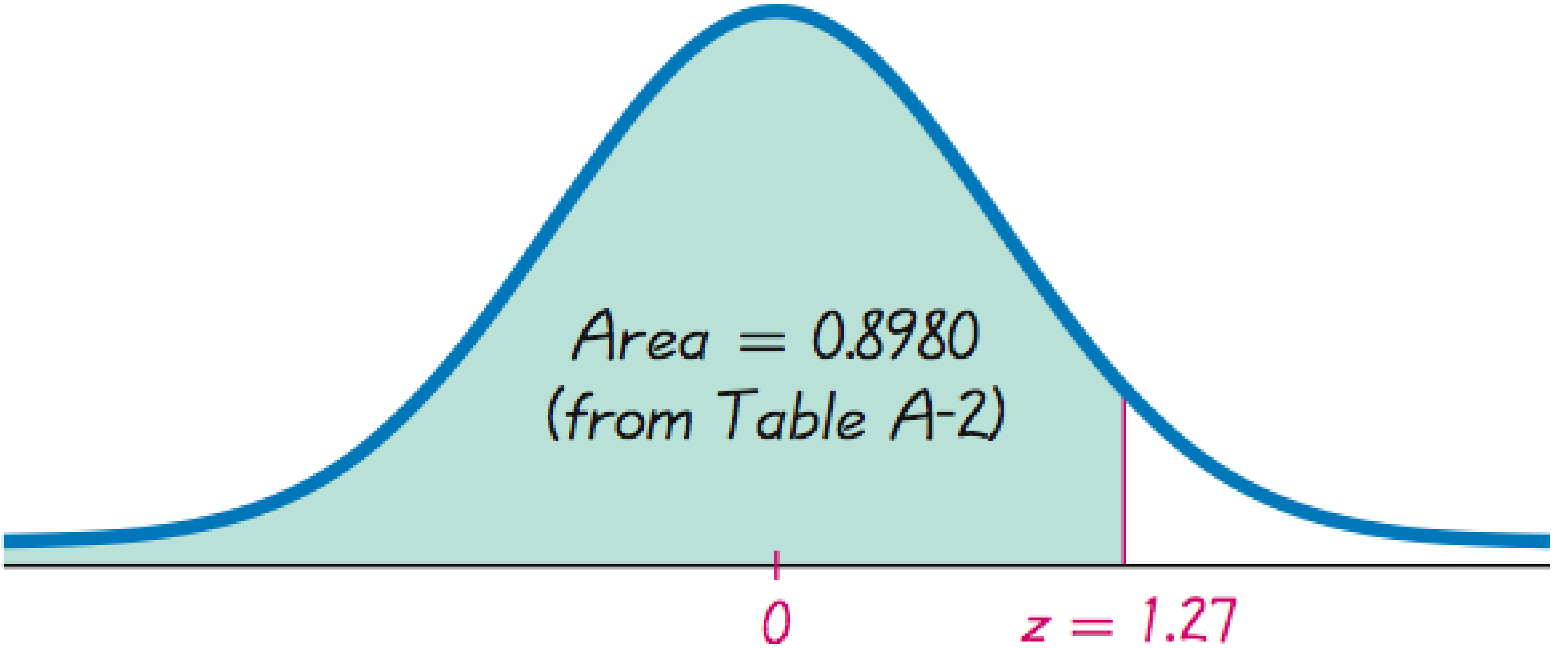
Example - cont

P (z < 1.27) = 0.8980



Or 89.80% will have readings below 1.27º.

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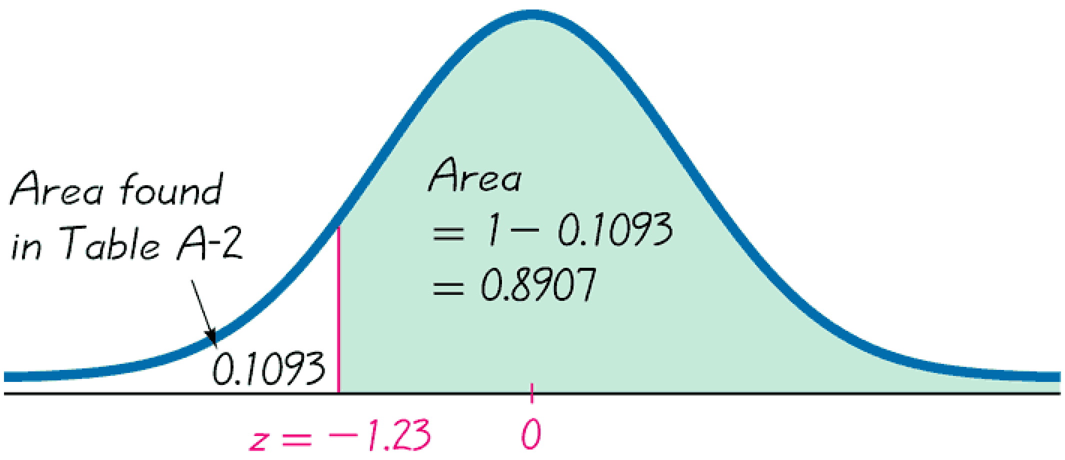
Example - Thermometers Again

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.

$$P(z > -1.23) = 0.8907$$



Probability of randomly selecting a thermometer with a reading above –1.23º is 0.8907.

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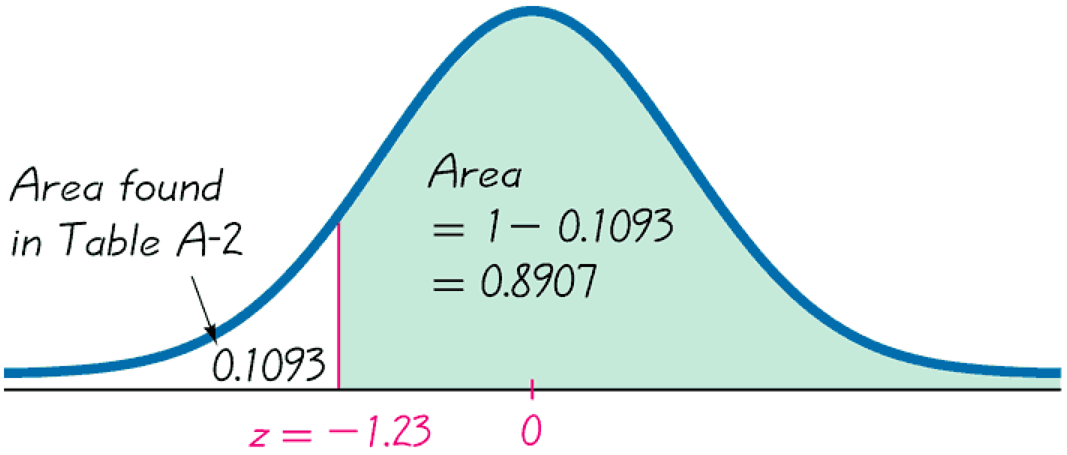
Example - cont

P (z > –1.23) = 0.8907



89.07% of the thermometers have readings above –1.23 degrees.

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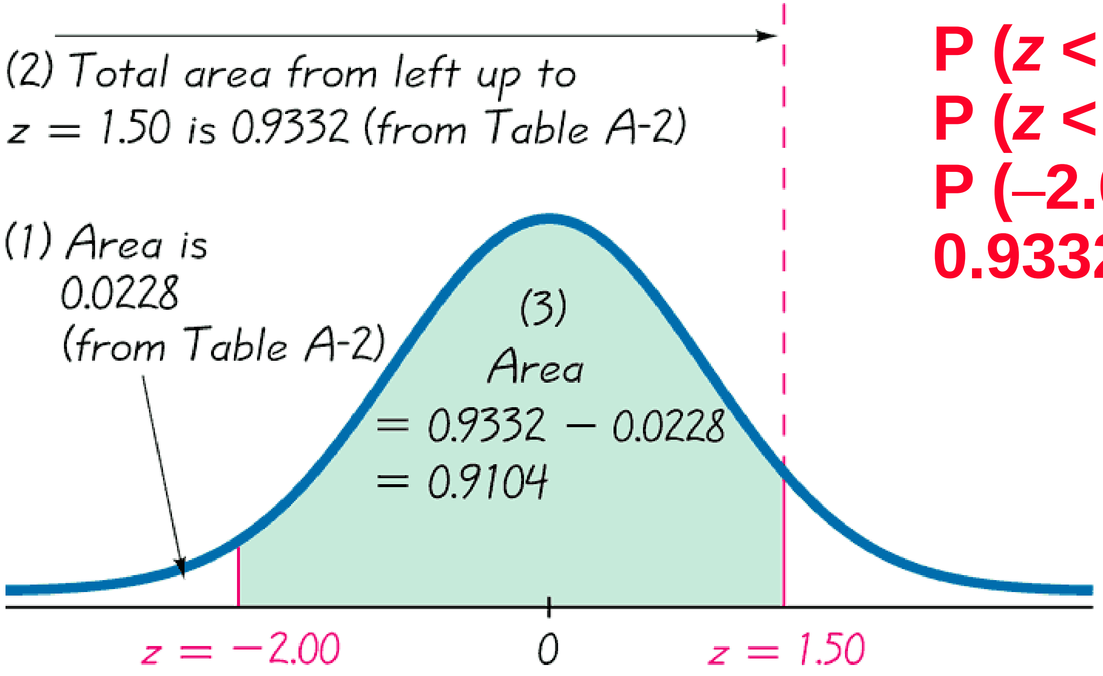
Example - Thermometers III

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



P (z < –2.00) = 0.0228
P (z < 1.50) = 0.9332
P (–2.00 < z < 1.50) =
0.9332 – 0.0228 = 0.9104

The probability that the chosen thermometer has a reading between – 2.00 and 1.50 degrees is 0.9104.

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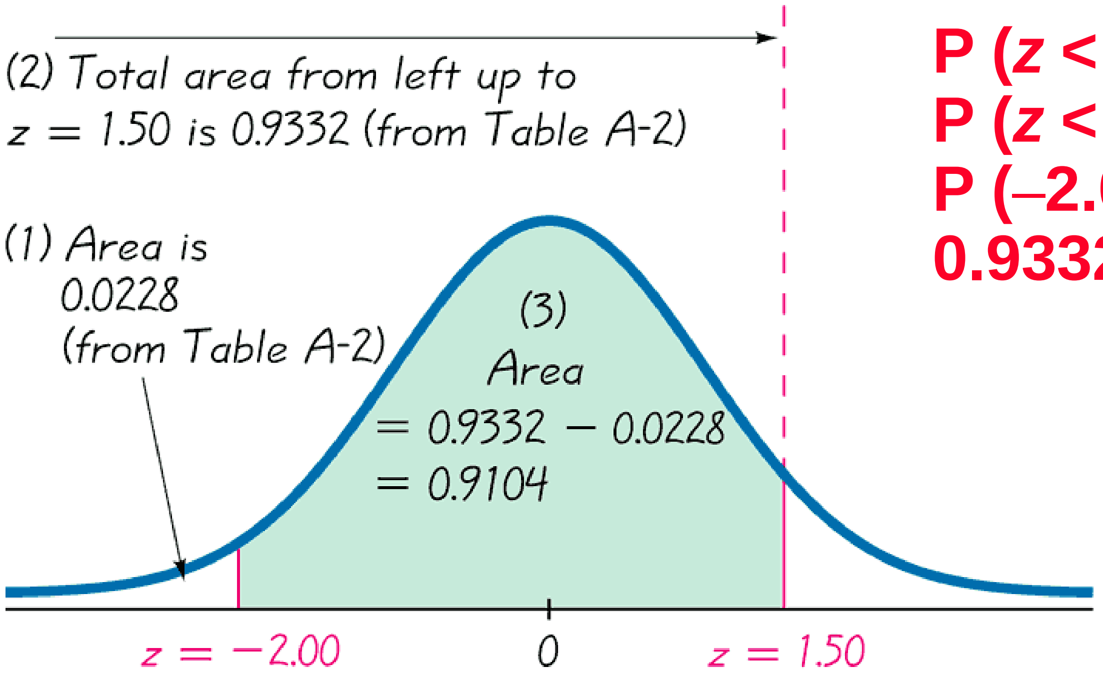
Example - cont

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



$P(z < -2.00) = 0.0228$
$P(z < 1.50) = 0.9332$
$P(-2.00 < z < 1.50) =$
$0.9332 - 0.0228 = 0.9104$

If many thermometers are selected and tested at the freezing point of water, then 91.04% of them will read between –2.00 and 1.50 degrees.

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Notation

$$P(a < z < b)$$

denotes the probability that the z score is between a and b.

$$P(z > a)$$

denotes the probability that the z score is greater than a.

$$P(z < a)$$

denotes the probability that the z score is less than a.

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Finding a *z* Score When Given a Probability Using Table A-2

1. **Draw a bell-shaped curve and identify the region under the curve that corresponds to the given probability. If that region is not a cumulative region from the left, work instead with a known region that is a cumulative region from the left.**

2. **Using the cumulative area from the left, locate the closest probability in the body of Table A-2 and identify the corresponding *z* score.**

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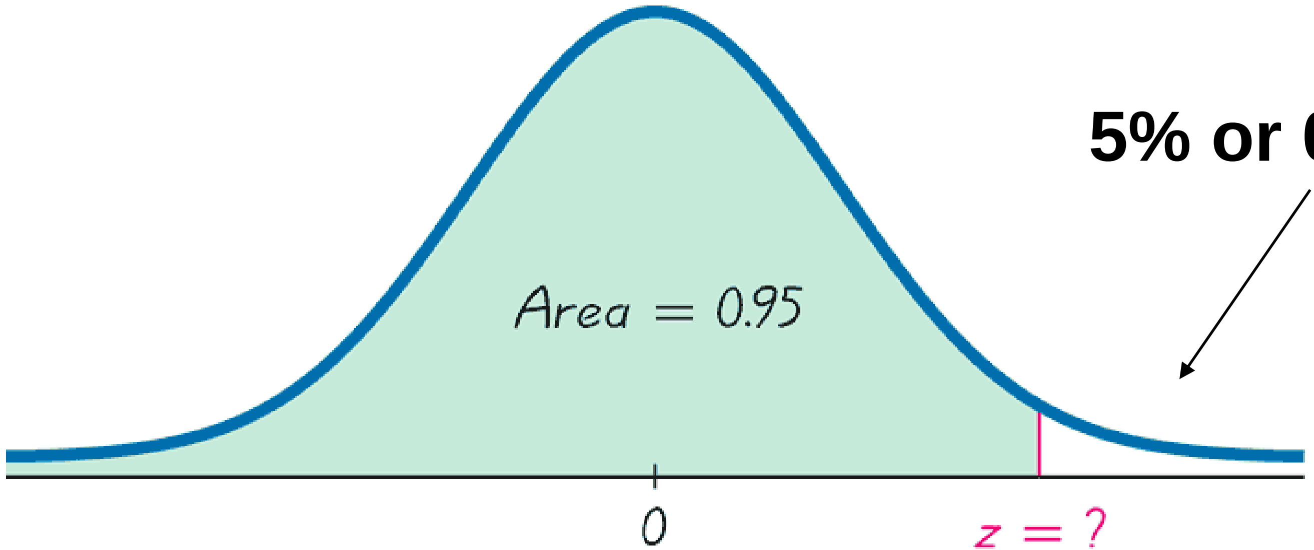
Finding *z* Scores When Given Probabilities

5% or 0.05



(*z* score will be positive)

Finding the 95th Percentile

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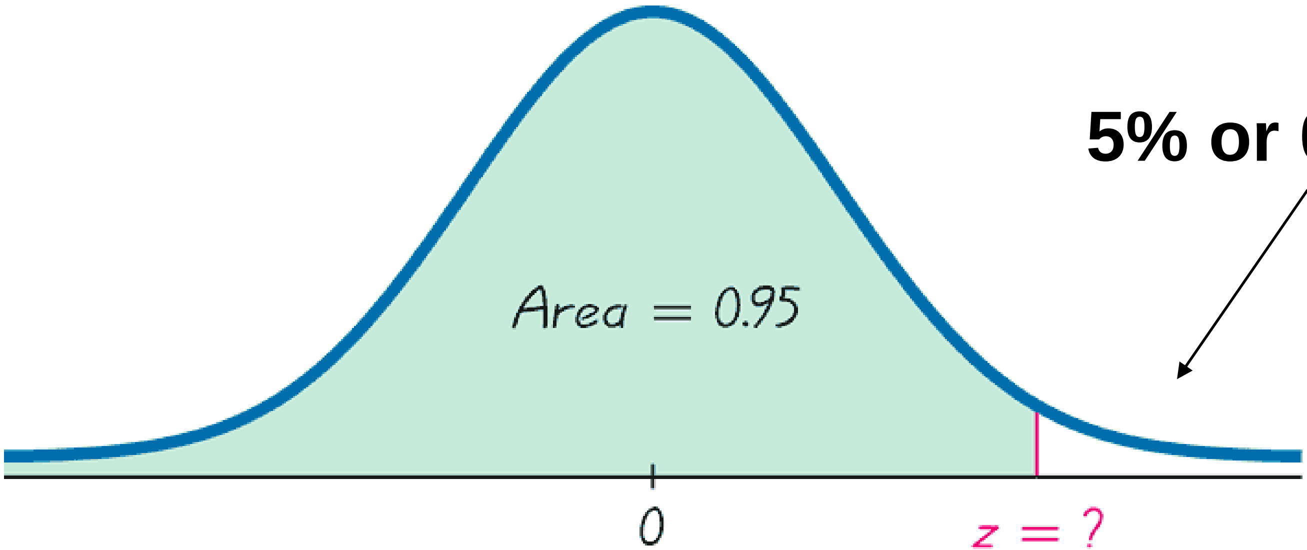
Finding *z* Scores When Given Probabilities - cont



5% or 0.05

(*z* score will be positive)

1.645

Finding the 95th Percentile

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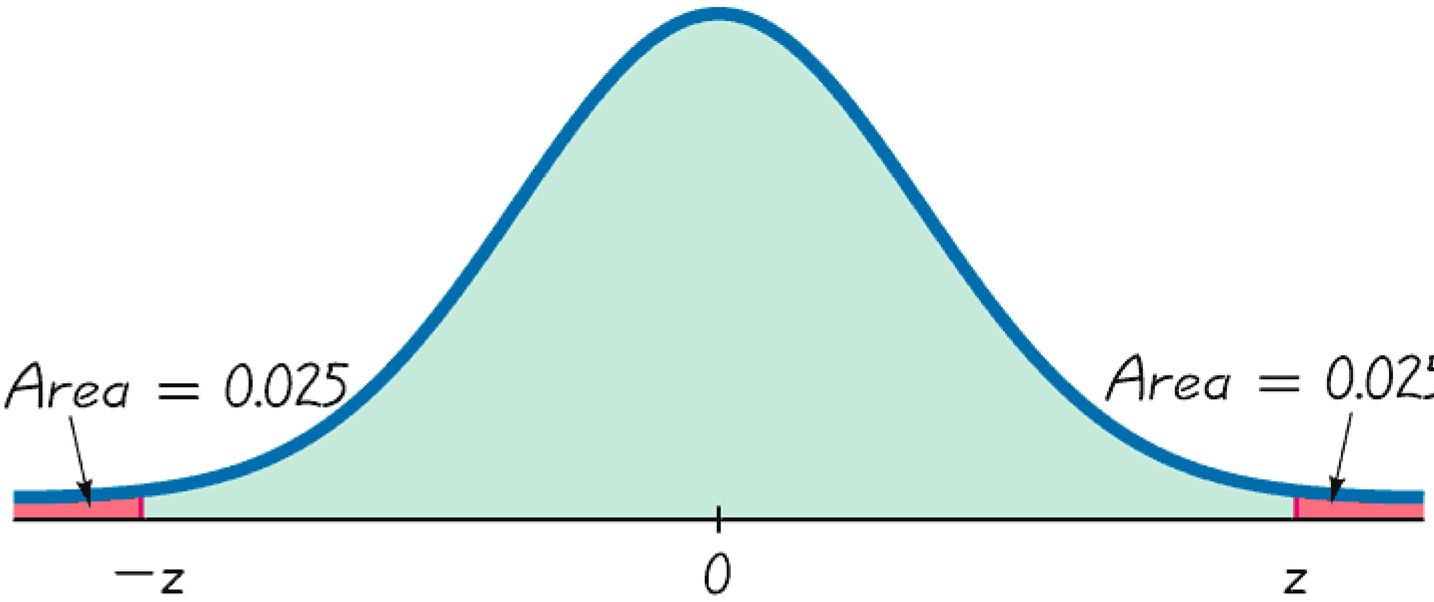
Finding z Scores When Given Probabilities - cont



(One z score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

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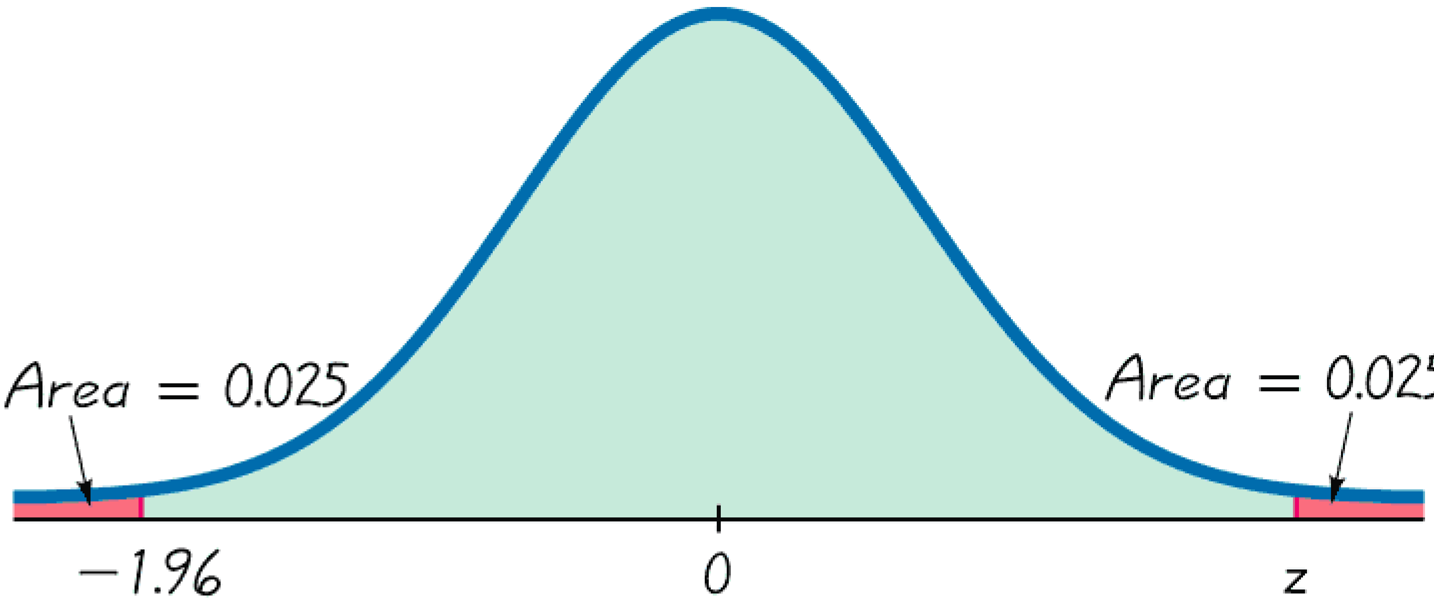
Finding *z* Scores When Given Probabilities - cont



(One *z* score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

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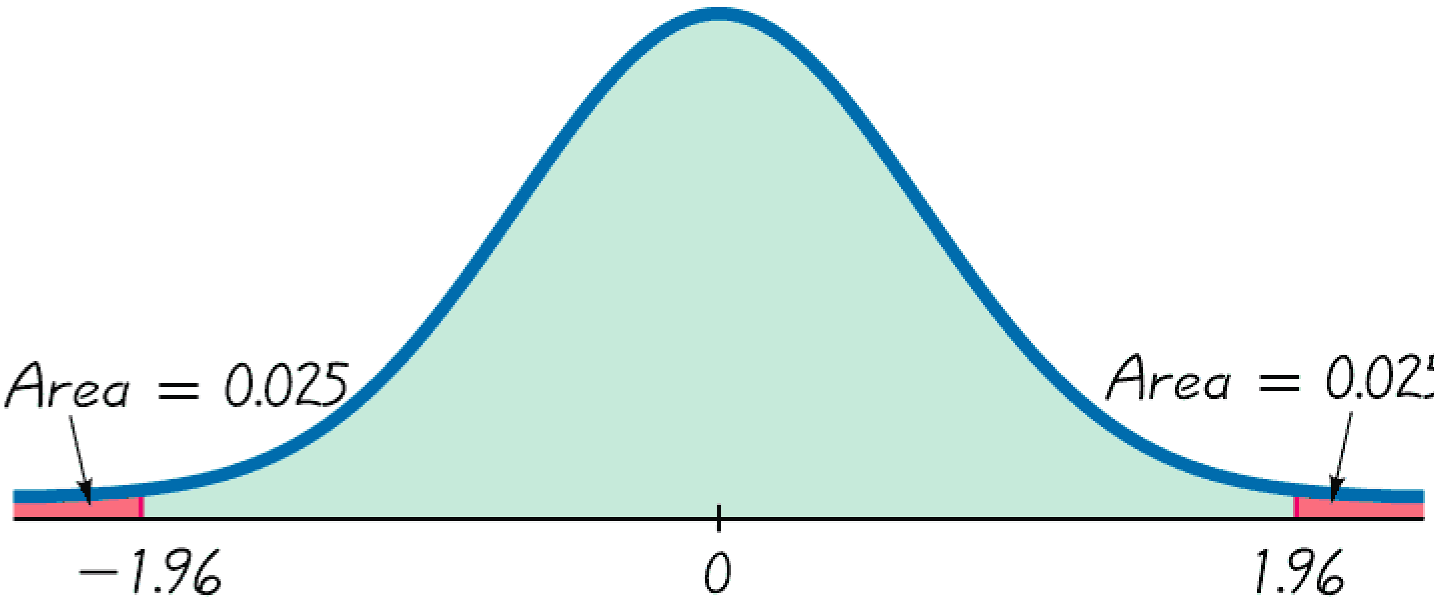
Finding *z* Scores When Given Probabilities - cont



(One *z* score will be negative and the other positive)

Finding the Bottom 2.5% and Upper 2.5%

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Recap

In this section we have discussed:

- **Density curves.**
- **Relationship between area and probability.**
- **Standard normal distribution.**
- **Using Table A-2.**

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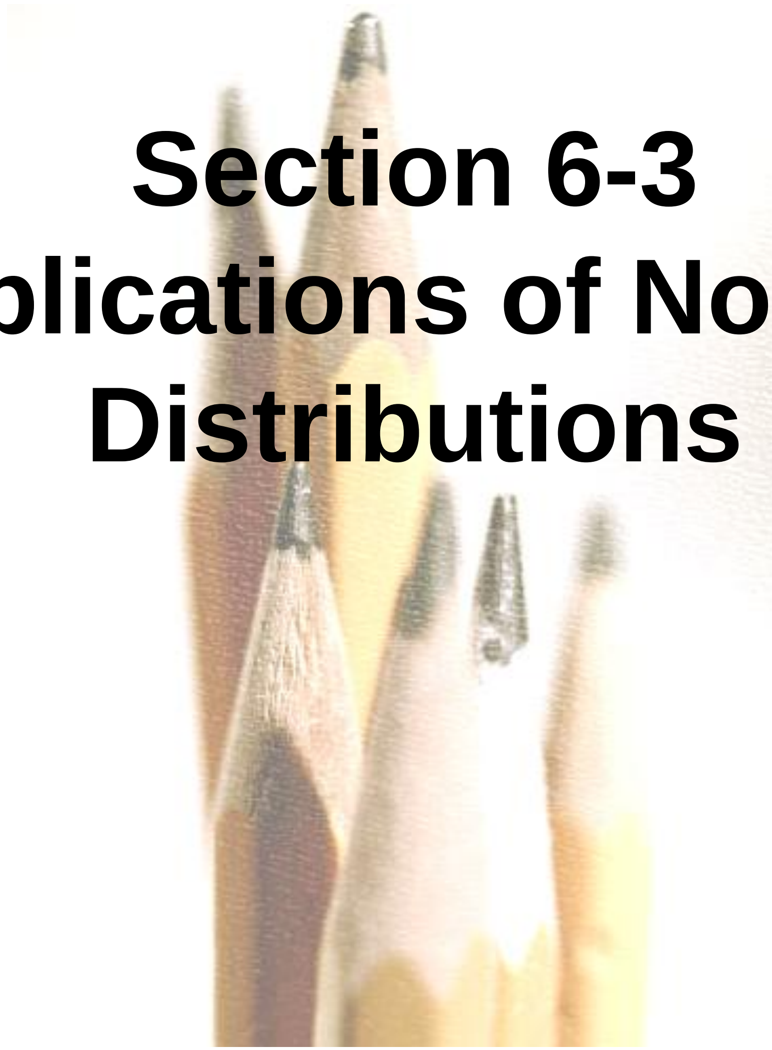
Section 6-3 Applications of Normal Distributions



Key Concept

This section presents methods for working with normal distributions that are not standard. That is, the mean is not 0 or the standard deviation is not 1, or both.

The key concept is that we can use a simple conversion that allows us to standardize any normal distribution so that the same methods of the previous section can be used.

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Conversion Formula

$$z = \frac{x - \mu}{\sigma}$$

Round *z* scores to 2 decimal places

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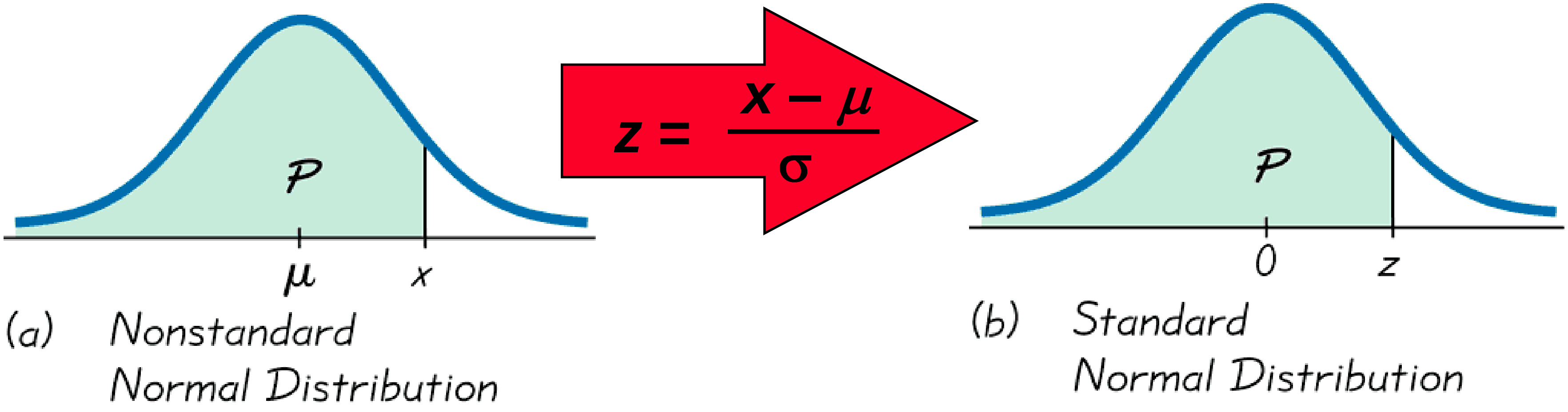
Converting to a Standard Normal Distribution



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Example – Weights of Water Taxi Passengers

In the Chapter Problem, we noted that the safe load for a water taxi was found to be 3500 pounds. We also noted that the mean weight of a passenger was assumed to be 140 pounds. Assume the worst case that all passengers are men. Assume also that the weights of the men are normally distributed with a mean of 172 pounds and standard deviation of 29 pounds. If one man is randomly selected, what is the probability he weighs less than 174 pounds?

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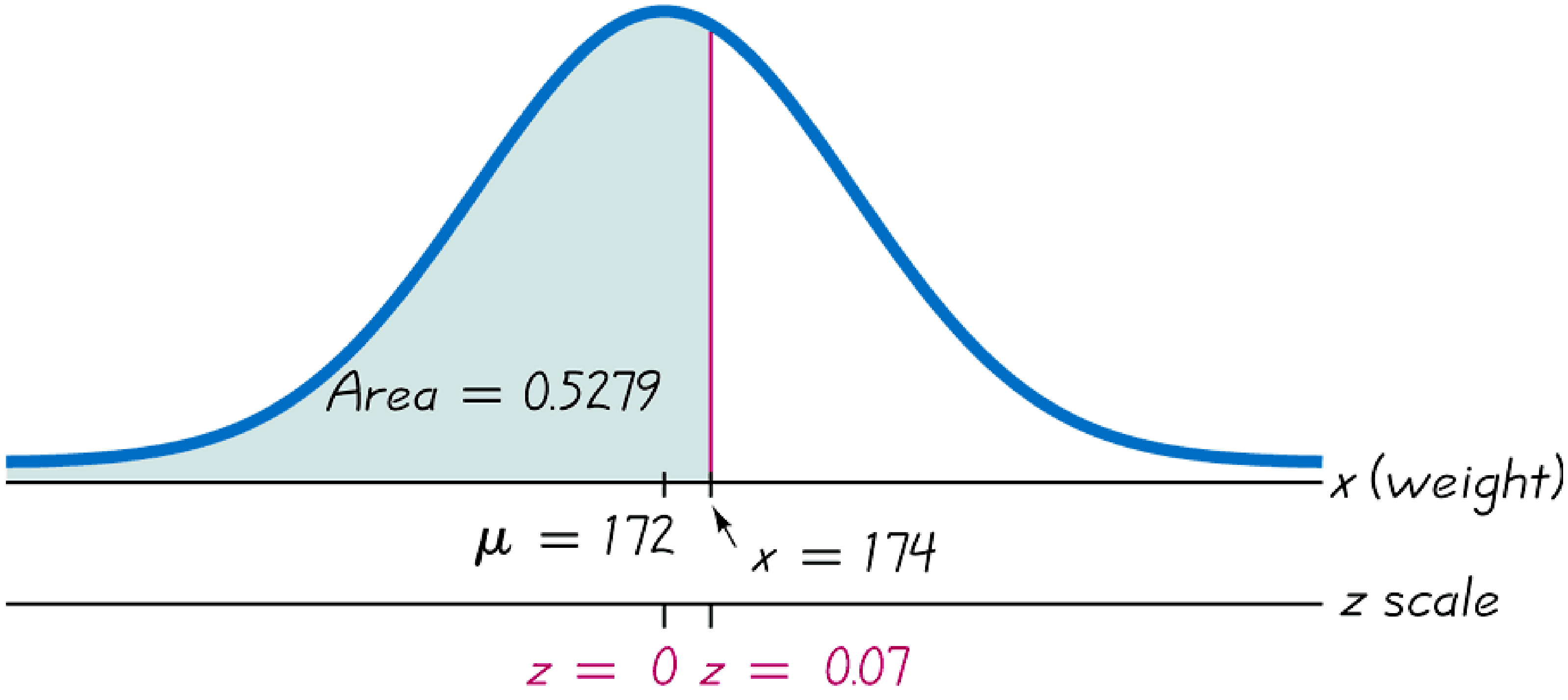
Example - cont

$\mu = 172$

$\sigma = 29$

$$z = \frac{174 - 172}{29} = 0.07$$



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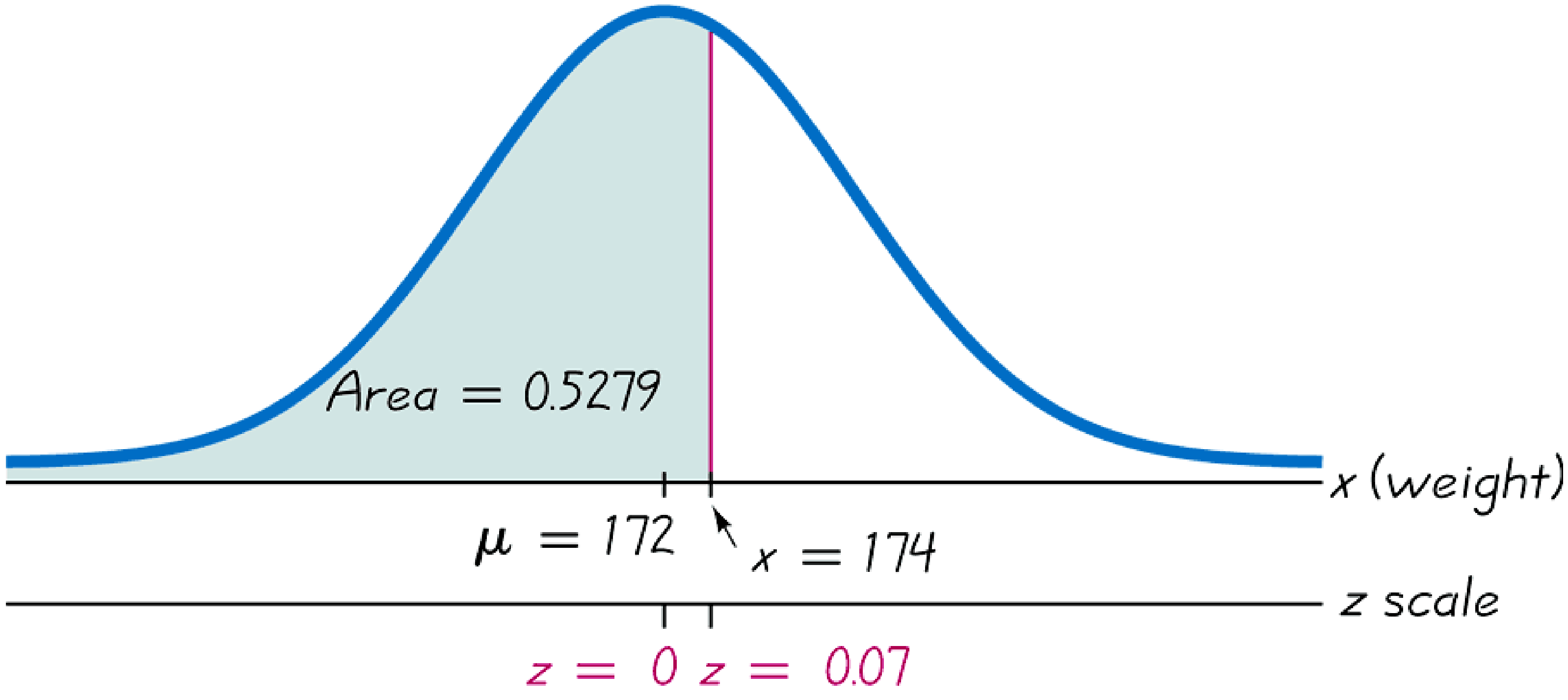
Example - cont

μ = 172

σ = 29

P (*x* < 174 lb.) = P(*z* < 0.07)



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Helpful Hints

1. **Don't confuse *z* scores and areas. *z* scores are distances along the horizontal scale, but areas are regions under the normal curve. Table A-2 lists *z* scores in the left column and across the top row, but areas are found in the body of the table.**
2. **Choose the correct (right/left) side of the graph.**
3. **A z score must be negative whenever it is located in the left half of the normal distribution.**
4. **Areas (or probabilities) are positive or zero values, but they are never negative.**

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Procedure for Finding Values Using Table A-2 and Formula 6-2

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the *x* value(s) being sought.

2. Use Table A-2 to find the *z* score corresponding to the cumulative left area bounded by *x*. Refer to the **body** of Table A-2 to find the closest area, then identify the corresponding *z* score.

3. Using Formula 6-2, enter the values for μ, σ, and the *z* score found in step 2, then solve for *x*.

   $$x = \mu + (z \cdot \sigma)$$ (Another form of Formula 6-2)

   (If *z* is located to the left of the mean, be sure that it is a negative number.)

4. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.

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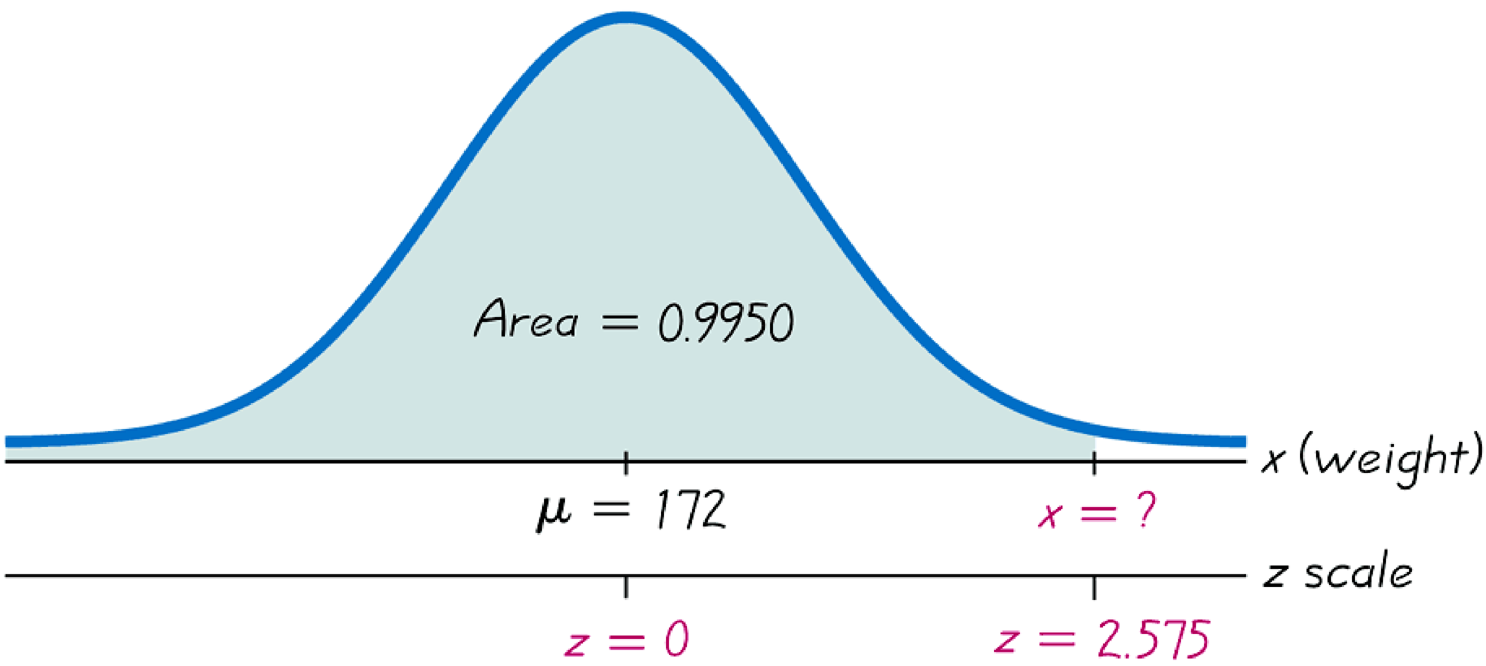
Example – Lightest and Heaviest

Use the data from the previous example to determine what weight separates the lightest 99.5% from the heaviest 0.5%?



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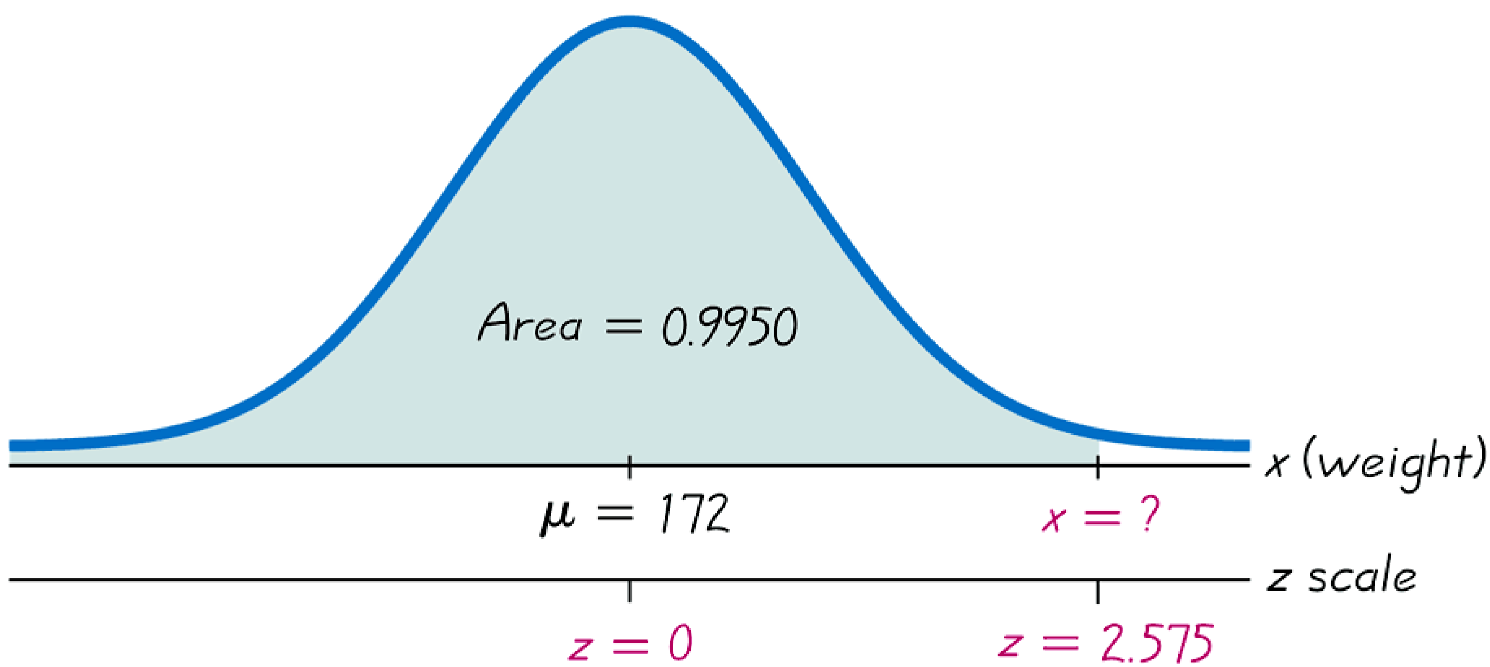
Example –
Lightest and Heaviest - cont

$$x = \mu + (z \quad \sigma)$$
$$x = 172 + (2.575 \bullet 29)$$
$$x = 246.675 \text{ (247 rounded)}$$



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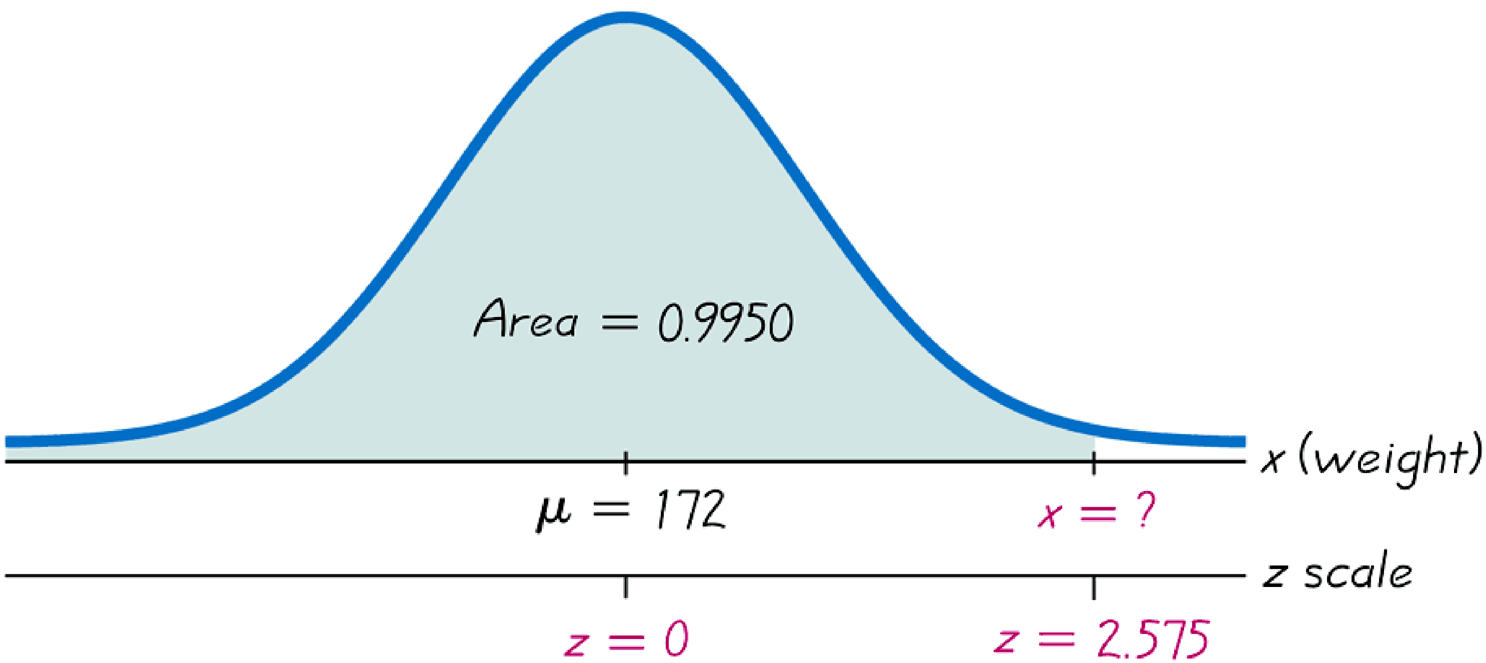
Example – Lightest and Heaviest - cont

The weight of 247 pounds separates the lightest 99.5% from the heaviest 0.5%



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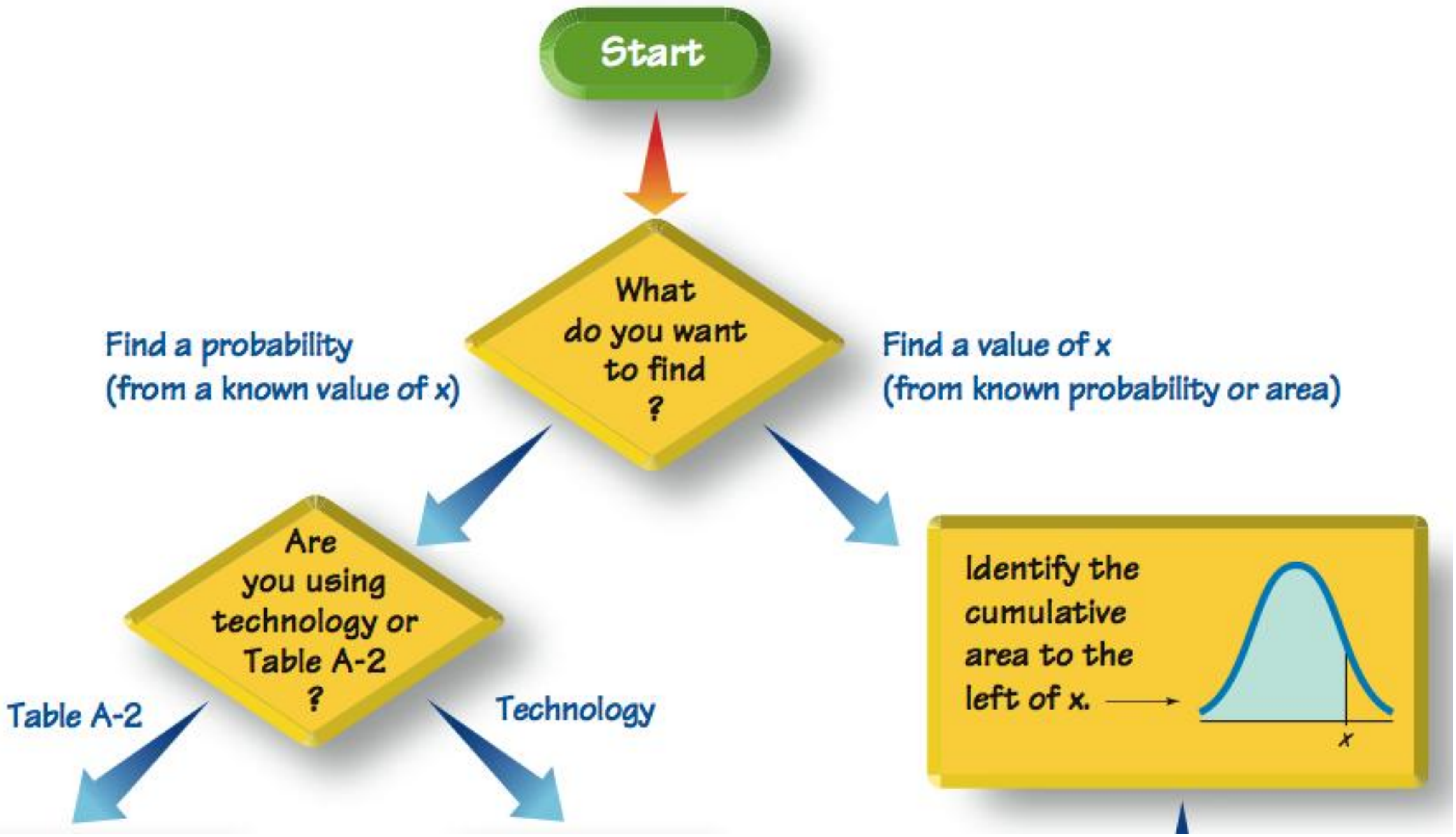
Applications with Normal Distributions



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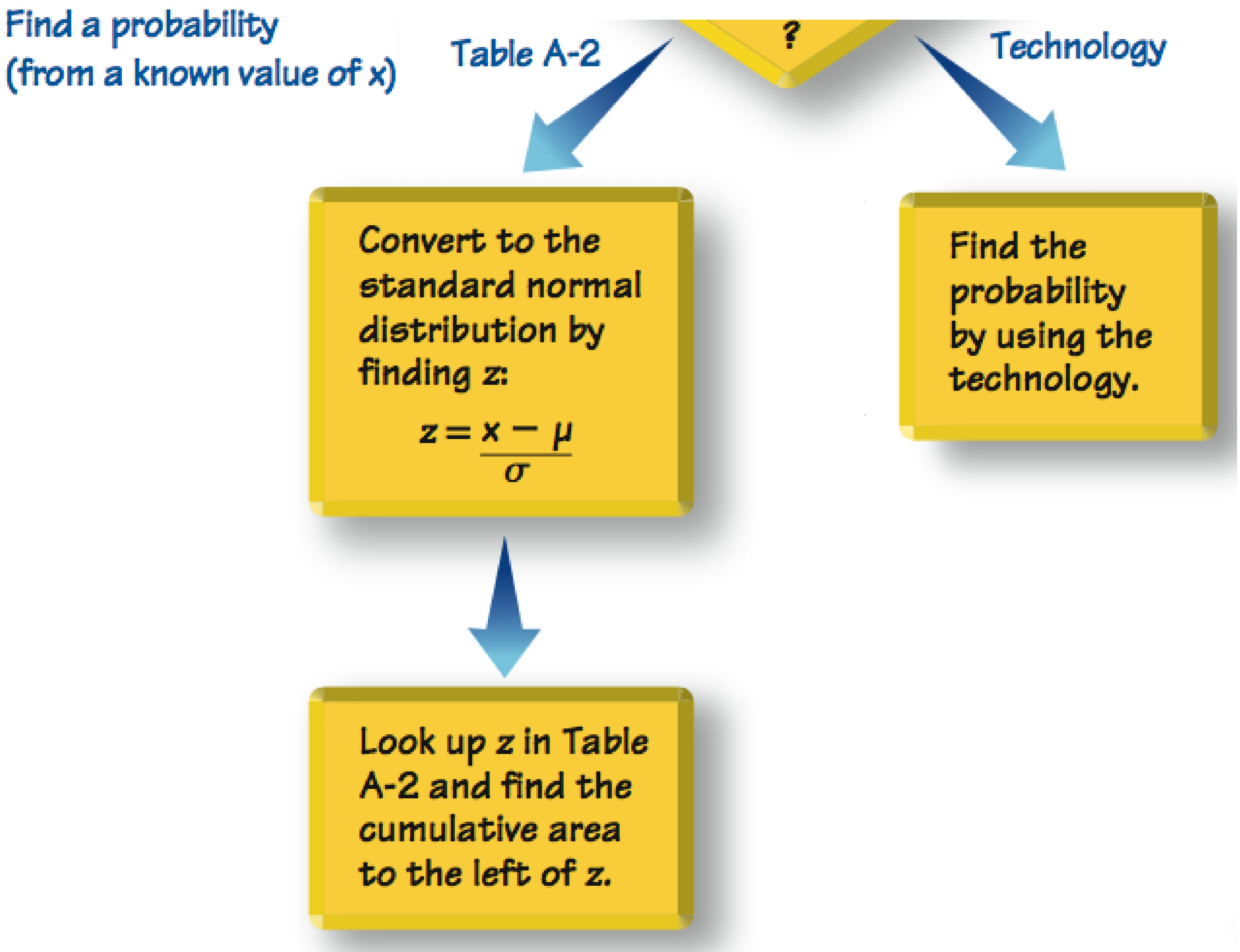

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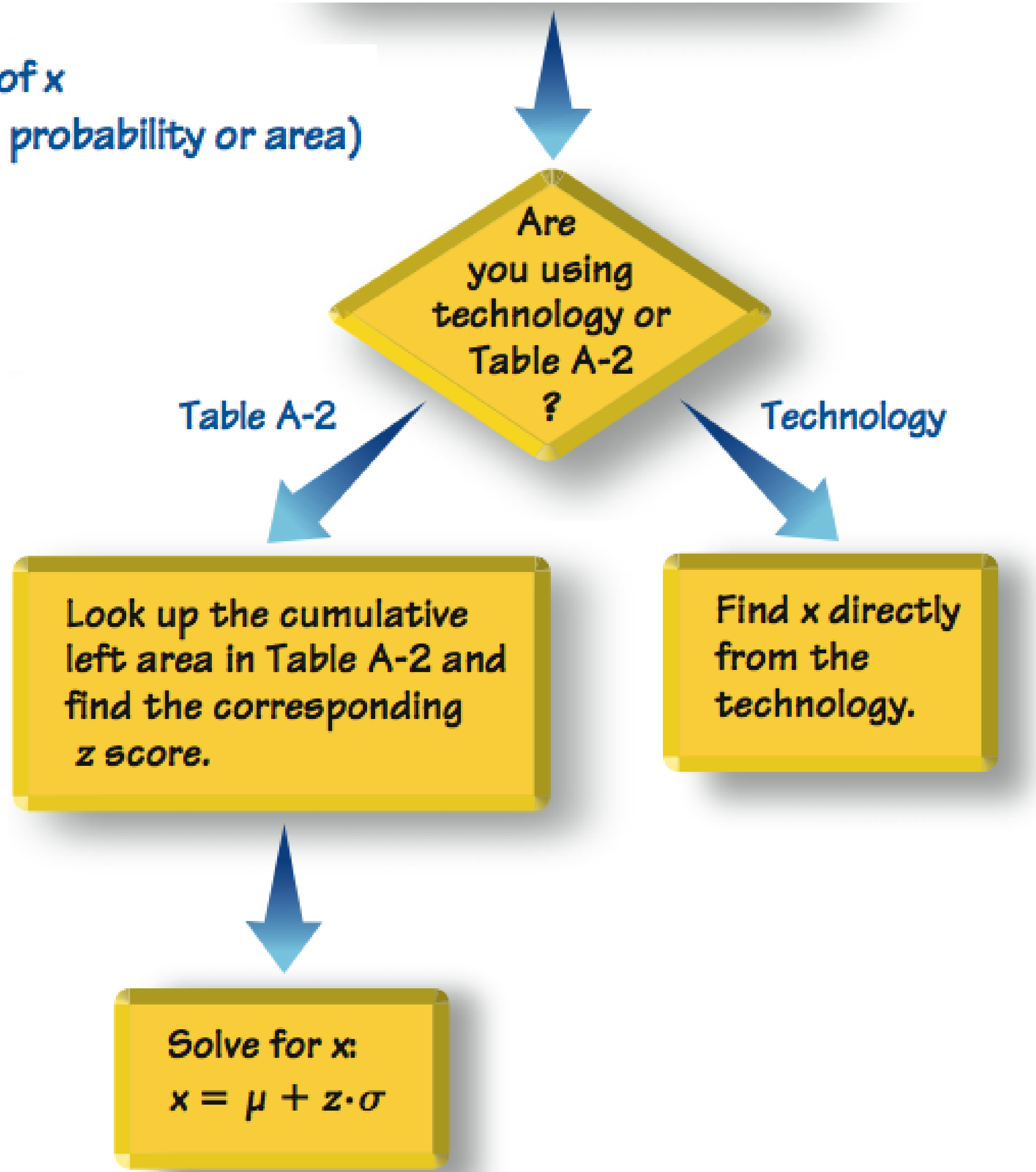
Find a value of x
(from known probability or area)



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Recap

In this section we have discussed:

- **Non-standard normal distribution.**
- **Converting to a standard normal distribution.**
- **Procedures for finding values using Table A-2 and Formula 6-2.**

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